Le valutazioni numeriche di collettività

Silvia De Marchi

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The following is my translation of excerpts from Silvia De Marchi's original 1929 article on numerical evaluations published in *Archivio Italiano di Psicologia*.

It is widely believed that the British physicist Lewis Richardson invented the method of magnitude estimation (*Journal of General Psychology*, 1928, **2**, 324-352). De Marchi (1929) anticipated Richardson in this invention. She applied the method to numerousness plotting the respective psychophysical function. Vittorio Benussi read De Marchi's paper at the *IV Congresso Nazionale di Psicologia* in 1923. The *Atti del IV Congresso Nazionale di Psicologia*, Firenze: Bandettini, 1925 (on title page), report a long abstract of this paper.

I thank Natale Stucchi for making me notice that De Marchi was an undergraduate in 1923 and that the *Atti del IV Congresso Nazionale di Psicologia* specifically report (p. 15) that Vittorio Benussi read De Marchi's paper to the audience. This information and the fact that De Marchi was Benussi's pupil suggest that Benussi was most probably the main source of ideas for De Marchi.

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Numerical evaluations of collectivities

(*with 23 figures*)

I. – Introduction

a) – *Preliminary establishment of facts*. – If we show to different subjects a complex of dots as that in fig. 1 (dots = 158) for a very short time, so to exclude the possibility that subjects count even only a part of the dots, and invite the subjects to *evaluate* with a numeral the seen collectivity, we can ascertain that one obtains evaluative judgments that are subjectively *confident* (in the evaluation, subjects oscillate for ex. from 70 to 80, from 120 to 140, from 300 to 320, but within these limits they are confident).

One can realize that in some cases the obtained evaluations are larger than the actual number of dots and in other cases are less than this number (1). In sum, one obtains overvaluations or undervaluations that are true and proper «*individual constants*» (2). Those for ex. who have once undervaluated a complex, undervaluate in front of new collectivities. Thus the under as well as the overvaluations correspond to typical qualities of subjects—they are not incidental phenomena but, as we have said, are individual constants (3).

(1) V. BENUSSI *Aus der forensischen Psychologie: Die Fehlerwurzeln unserer Aussagen*. (*Der Friede 40*, Pp. 323 and foll. Vienna, 1928).

(2) H. A. NANU – cited by K. Marbe in: *Grundzüge der forensischen Psychologie.* München 1913, – noticed only the undervaluator type, probably for the fact that among subjects in general, as it also results from our experiments, the undervaluator type is more frequent.

(3) C. L. MUSATTI (*Analisi del concetto di realtà empirica*. P. 134. Città di Castello, 1926) interprets these «constants» as due to a *different meaning* that the various elements of the numeric series would have for the different subjects in relation to their different experience of «counting».

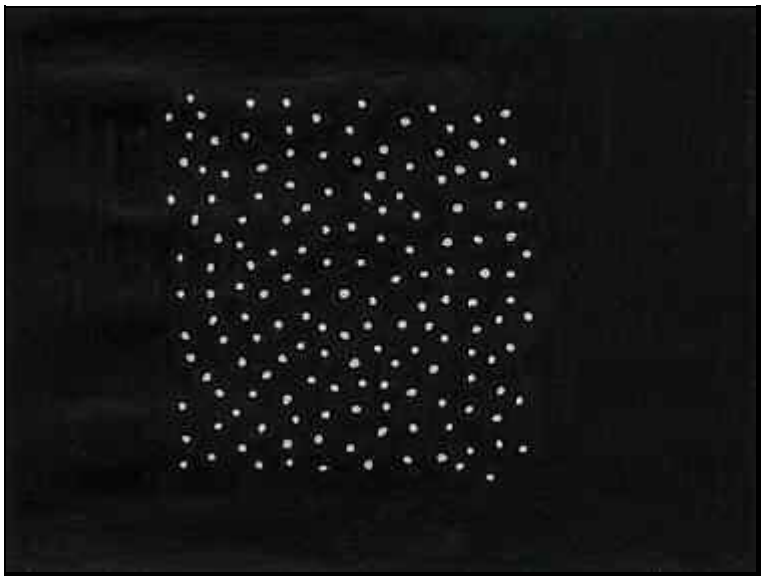

Fig. 1

[*Translator's note*: Fig. 1 appears at the top of the original p. 178. The text of p. 178 is not translated.]

[*Translator's note*: the part of the original text from p. 178 to this point of p. 181 is not translated.]

The present study has thus the purpose of analyzing the *factors of evaluation* (1).

Evaluative situations result from multiple determinants, of which some are *external*, that is, are given by the objective conditions of the stimuli, and others are *internal*, which constitute the conscious behavior of the subject in front of these stimuli. Among the *external determinants* one may consider:

1) the duration of exposure;
2) the size of the surface occupied by the single collectivities;
3) the density of the exposed elements (dots);
4) their spatial disposition;
5) their disposition in time, depending on whether they are successive or synchronous;
6) the disposition in space and time, depending on whether the collectivities are at rest or in motion;
7) the disposition in space and time, depending on whether their succession is rapid or slow;
8) depending on whether their motion is rapid or slow;

(1) Some results of these my researches were presented in a preliminary note at the IV National Congress of Psychology (1923); cfr. Atti del IV Congresso Nazionale di Psicologia – Firenze 1926, pp. 131 and foll.

among the *internal determinants*:

1) the attentional behavior: a) attentional setting - b) attentional concentration or attitude;

2) mental fractionation;

3) reflexive elaboration;

4) absolute impression of multitude or poverty;

5) immediate or mediate formal connections;

6) the insistence of consecutive mental presence.

In sum, *to analyze the evaluative process* means to specify the importance that all these factors have for the result of an evaluation. This process must be distinguished from that that can lead automatically to learn to evaluate adequately from the knowledge of one's own evaluative errors. The analysis of this second process would constitute not «the psychology of evaluations» but «the psychology of learning of a particular behavior adequate with reality» (1).

Here we instead consider the laws of evaluative errors in the case in which one has not yet learned to evaluate adequately and it is not possible that the subject learns because he has not ever had knowledge of his errors. Having posed the general theme this way I now pass to the technical aids.

c) – *Devices*. Two devices were used: one for evaluations of collectivities at rest, for *static experiments,* and the other for evaluations of collectivities in motion, for *dynamic experiments*.

For the static experiments, an Ernemann projection apparatus was used whose objective, movable within large limits, allowed for projections having side length of some meters or few centimeters. The size of the surface of the collectivities was varied by varying the position of the objective, that is, by varying the distance between the slide and the transparent screen on which the collectivities to be evaluated were projected.

A kimograph, rotating at a uniform speed, had an insulating mantle in which slits of different width were cut out; a metallic pen

(1) In the psychological laboratory of Padua, experiments are in course with the purpose of determining the influence that the knowledge of one's own evaluative errors exercises on the evaluative processes.

was in contact with the mantle in such a way that it could touch, through one or the other slit, the metallic part of the kimograph for different times corresponding to the different slit widths. The lamp of the projection apparatus was activated by a circuit that passed through the metallic pen and the kimograph so to obtain illuminations of the slide of different durations; the duration of these projections could also be varied by varying the rotation speed of the cylinder of the kimograph. One could thus obtain exposures of 200, 400, 8000 σ or more.

In the experimental room, illuminated in twilight, the kimograph was put in motion and an agreed signal was given some seconds before the exposure of the figure (attention!) and immediately before the exposure a second signal was given which indicated the imminent appearance of dots (now!).

The subjects had the task to look attentively at the screen paying attention to the screen in a uniform manner. The procedure of the experiment will be described later below.

[*Translator's note*: the remaining part of the original p. 183 is not translated.]

[*Translator's note*: the previous part of the original p. 184 is not translated.]

d) – *Method*. In the presentation of the collectivities of dots the tachistoscopic method was followed. Exposures must in fact be very short to prevent subjects from arriving at the solution of the task by «counting» the elements of all or part of the collectivity. If he could do this one would no longer have a numerical evaluation, the process by which a perceived aggregate is expressed by numerals in conditions that exclude any possibility of numbering its elements.

With respect to the behavior of the subjects, the experiments were distinguished in: *purely evaluative* and *introspective* (1).

In the first case the subject had only to put down on record the first, thoughtless evaluation that arose in his mind as soon as the projection was seen, without stopping on possible posterior evaluations

(1) The experiments were all collective.

that were based on impressions elaborated after, nor on particular elements of the subjective behavior in the phases preceding the writing of the record.

Instead in the second case the task of the subjects was to write down a minute introspective record about the immediate evaluation, about the phases that preceded it, about mental elaborations following the first evaluation, and about possible new evaluations arisen from reflection, comparisons with previous experiences, etc.

This second group of experiments was carried out after the first so that subjects, made familiar with their experiences, were ready to analyze their own internal behavior and to specify those introspective internal data that less expert subjects would not be able to notice. These two methods of *immediate pure evaluation* and of *reactive introspection* allow us to process separately the obtained data; first the evaluative numerical data concerning the different internal and external conditions of the experience of pure immediate evaluation, then the introspective data which help us integrate the numerical-evaluative results in their different conditions of experience. In the present work we consider principally the results of immediate numerical evaluations, limiting ourselves to those hints to introspective data that become necessary for the interpretation of quantitative data. A more exhaustive processing of introspective data will be the subject of a second study.

Before these two groups of experiments, two series of experiments on «*absolute impressions of collectivity*» were carried out.

d) – *Previous researches*. Before reporting the results obtained from our experiments I consider briefly few data of previous researches. These data are due above all to Liebenberg (1) and to Benussi (2).

Liebenberg (and more recently *Mokre*) (3) searched the laws of evaluation for very small collectivities; the material of *Liebenberg*

(1) R. LIEBENBERG: *Ueber das Schätzen von Mengen*. – Zeitschrift f. Psych. 68 – 1914 – Pp. 321 and foll.

(2) B. BENUSSI: Op. cit..

(3) H. MOKRE: *Ueber den Einfluss von Grösse und Abstand auf die Mengenauffassung* (Zeitschrift f. Psych. 105 – Pp. 195 and foll.) 1927.

does not exceed 18 units; these are disposed on horizontal or vertical lines or on curves; are of different color or grouped three by three or four by four. He ascertains that collectivities from 5 to 7 are evaluated exactly if they are exposed tachistoscopically, while the other collectivities are evaluated the more the more the number of elements increases. Considering these experiments we may conclude: *a*) that the situation studied by *Liebenberg* is not a true situation of evaluation, but is similar to that of «counting», *b*) that one cannot obtain with his procedure a pure evaluation because it is based on exercise. We instead, as we have said, are not interested in searching the situation of the mnestic learning of a relation between impression of collectivity and numeral, but in *the spontaneous correlation* based on one hand on the pure impression of collectivity and on the other hand on numerical impressions.

It is also evident that, when there are few elements, it is also possible to count them during the consecutive impression that follows every perception, should it not be possible in the passing moment when they are exposed. The collectivity remains, even some time after it disappearance, «*mentally present*» (1) and in front of this mental presence we can behave as if in front of a *perceptually present* object.

When dots are lined up on a straight line or a curve, experience also teaches us that dots tend to be subjectively grouped in complexes of three or four. It is probable that the number of elements given objectively is overvaluated due to the fact that partial enumerative processes enter the evaluation. In general *Liebenberg* also did not analyze in any particular way the importance of the factors of shape and movement.

The value of structured elements was instead noticed by Benussi (from whom I took the examples shown in a). The present researches are related to his experiments and try to develop analytically some points of view that had been only pointed out.

(1) *Mental presence and perceptual presence* in the sense used by BENUSSI: cfr.: «La suggestione e l'ipnosi come mezzi di analisi psichica reale». (Zanichelli, Bologna – 1925 – pp. 10-14).

2. – **Absolute impressions** (of *vf, f, i, m, vm*)

The target of the evaluations that now we have to analyze is a «magnitude». As such it is subject to the influence of the impressions of *absolute magnitude* and to the law of relativity of the impressions of magnitude in general. Let us try to specify the meaning of the words «impression of relativity» and «absolute impression»: we have the first case when we are in front of objects that, compared with one another, give the impression of «larger» «smaller» «equal». The same objects can however give sometimes also the impressions of «large» «small» without being based on references to an ideal unity of measurement nor to any term of comparison. In this case we are in front of an absolute impression of magnitude; and all objects that have magnitude can evoke absolute impressions of magnitude in us, independently of any comparative attitude (1).

Before considering the experiments on numerical evaluations let us present some data on *absolute impressions*. The perception of a collectivity can enliven absolute impressions of *large*, or *small*, and is also possible that numerical evaluations are founded on such an absolute impression. It is possible for ex. that the under and over-valuator types found their valuations on opposed absolute impressions of collectivity.

Such absolute impressions depend on the general set of the subject. In fact if for ex. the set of the subject is given by the representation *newspaper*, and if the subject is shown a newspaper of 80 cm

(1) It is perhaps useful to mention what was observed on absolute impressions in other perceptual fields. It is sufficient to note that *absolute* qualitative impressions are given particular emphasis by L. MARTIN and G. MÜLLER in connection with the evaluation of weight (Zur Analyse der Unterschiedsempfindlichkeit. Leipzig – Barth – 1899 – pp. 43-50), by W. STERN in connection with the evaluations of time and space (Ueber Schätzungen etc. insbes. Zeit u. Raumschätzungen in Beiträge z. Psych. Der Aussage – Leipzig – 1904), by V. BENUSSI in connection with the evaluations of duration, brightness, and dissimilarity (Zur experimentellen Analyse des Zeitvergleichs. – I Zeitgrösse und Betonungsgestalt, Heidelberg 1913 – p. 13 and foll.).

height, this newspaper will give to the subject an absolute impression of «large»; if instead of a newspaper one presents the surface of a table 80 cm long the subject will have an absolute impression of «small».

In our experiments, given a surface and particular width of the contained dots or small disks, it is evident that, depending on the ratio between the sizes of dots and surface, the impression of «many, few, very few, etc.» that can be obtained in front of a constant number of element will change.

We must thus determine: *a*) the *objective* number of dots that gives rise for a given surface to an immediate impression, pure from any other situation, of «many» «very many» «few» «very few» «indefinite»; *b*) the number of dots that a subject thinks is mentally necessary so as to have an impression of very many, many, indefinite, few, very few (*vm, m, i, f, vf*).

A – In the first experiment the subjects (8 in number) were presented, for times equal to 280 σ, with collectivities of dots arranged with *different density* on a *constant* surface in different exposures, and were given the task to evaluate generically the exposed collectivity with the following expressions: dots are *very many* (vm), or dots are *many* (m), or *indefinite* (i), or *few* (f), or *very few* (vf).

Twenty-three collectivities of dots, which varied from 5 to 162, were exposed to the subjects 8 times. On the basis of the obtained results, for each subject a table as table 1 was constructed in which the percentage of evaluations (vm, m, i, f, vf) is reported in correspondence with the objective number of dots.

On the basis of these tables, which were thus relative to *all* subjects, table 2 was constructed which reports the evaluation (in vm, m, i, etc.) whose percentage is the highest average among all subjects.

Integrating and rectifying the obtained data by the usual methods, from table 2 it results that, with negligible oscillations, a collectivity of 7 dots or less is judged to be composed of *very few* dots (vf), that a collectivity of 10 dots is in between very few and few dots (f - vf); that is considered to be composed if *few* dots (f) one that in reality has 15; that is considered as intermediate between few and

TABLE 1.

Obj. N.	vf	f	i	m	vm
5	100 %				
7	100 »				
9	25 »	75 %			
11	50 »	50 »			
13	25 »	75 »			
15		87.5 »	12.5 %		
17	25 »	75 »			
19		37.5 »	62.5 »		
21		25 »	62.5 »	12.5 %	
23		50 »	50 »		
27		25 »	62.5 »	12.5 »	
32			62.5 »	37.5 »	
37		25 »	62.5 »	12.5 »	
47			50 »	37.5 »	12.5 %
52				25 »	75 »
57				75 »	25 »
62				50 »	50 »
72		12.5 »		75 »	12.5 »
82				62.5 »	37.5 »
102					100 »
122					100 »
127					100 »
162					100 »

TABLE 2.

Obj. N.	vf	f	i	m	vm
5	+				
7	+				
9		+			
11	+				
13		+ ?			
15		+			
17		+			
19		+			
21			+ ?		
23		+ ?			
27			+		
32				+	
37		+ ?			
47				+	
52				+	
57					+
62					+
72				+	
82					+
102					+
122					+
127					+
162					+

indefinite (f - i) one that is composed of 20.5; that is *indefinite* (i) a collectivity given by 27 dots; is intermediate between indefinite and many (i - m) a collectivity of 45 dots; while one considers to be composed of *many* (m) one constituted of 57 dots; appears indefinite between many and very many (m - vm) that which is composed of 67 dots and, at last, appears composed of *very many* dots (vm) that which is constituted of 102 or more dots.

Grouping graphically these data we have the diagram of fig. 7.

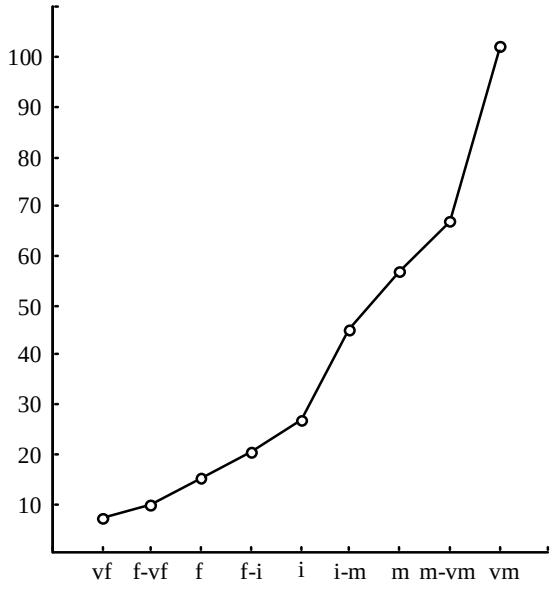

Fig. 7

We now observe:

We can consider the numerical values corresponding to the evaluations vf (7), f (15), i (27), etc. as constituting a geometric series in which the value 1 is given to the group that determines with frequency of 100 % the impression of collectivity formed by very few dots (vf): this fact allows us to determine the connection between the single absolute impressions, when two of these are given (for ex.: vf and f). The other impressions can thus be calculated independently of any experience. If we now consider the two extremes, *vf* and *vm*, we see that in *i* we have a quality that, subjectively, is equally distant from *vf* and from *vm*; so the quality *f* is equidistant from *vf* and *i*, and the quality *m* is equidistant from *vm* and *i*. A succession of subjectively equidistant impressions thus corresponds, in the objective conditions which determine these impressions, a suc-

cession of «stimuli» that results to be ordered according to a geometric series. This is a general law in psychophysics and our demonstrations are nothing ultimately but a particular case of this law.

The material of our experiment on absolute impressions is too limited for us to determine the under and overvaluator types, but we can for now suppose that such absolute impressions remain invariant for each different type.

From the obtained results one can instead note immediately that subjects are distinguishable in two types: *a*) that which prefers the evaluation «indefinite» (and this may be considered as a non-compromising solution of the task), *b*) that which, avoiding the evaluation «indefinite», prefers the others and is that is more confident in determining definitively the received impression.

The distinction of these two *types* will serve us in the interpretation of the data of the following experiment.

B – In this experiment we have a task opposite to that of the previous one. In the previous one the subject was invited to evaluate, with the expressions few, very few, many, etc., collectivities of elements of which he ignored the number. In this experiment instead he is invited (once he is shown a given surface and the size of the elements) to tell how many of these elements would be necessary to give the impression of many, very many, few, etc.

The experiment was carried out according to this scheme:

A dark surface delimited by luminous straight lines was projected on a screen. Only one dot was visible on the surface. The surface and the dot were constantly visible to the subject. Then the subject was asked the following questions in random order: «How many should the dots of this surface be to be *very few*?» «To be *many*?» «To be *few*?» «How many to be intermediate between very few and few, between indefinite and many, etc.?». The entire series of questions was presented twice. The subject put down on record each time the numeral that seemed to correspond to a subjectively satisfying solution of the task. The subjects were the same 8 subjects of the previous experiment.

While I reserve to another occasion the analysis of the reaction times necessary for the different subjects to solve the task, I pass to the obtained data collected in table 3.

TABLE 3.

Abs. Ev. in:	B	Bb	A
vf	7.9	4.85	7
f - vf	12.25	13.7	10
f	10.6	10.9	15
f - i	30.2	23.7	20.5
i	47.3	38.8	27
m - i	54.7	51.2	45
m	80.4	61.1	57
m - vm	101.8	74.3	67
vm	113.8	102.7	102

In it I indicate with *B* the overall mean data, with *Bb* the mean data relative to those subjects who showed in the previous experiment to be of type b), with *A* the data relative to the previous experiment (A). The data of the table give us the curves of the diagram in fig. 8.

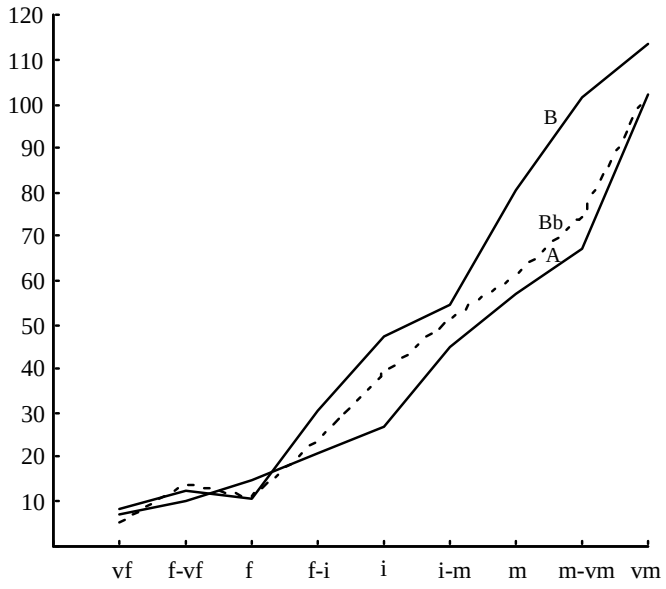

Fig. 8

From these data it results: 1) that in the experiences B the numeric values corresponding to the expressions *f, i,* etc. grow more rapidly than in the experiences A; 2) that, if in the experiences B one neglects the subjects of type *a*) (that is those who prefer evaluations of the kind *indefinite*) to take into consideration only the subjects of type *b*), one has data that are slightly distant from the data of the experiences A.

However we can also observe that, for the subjects of type b), the numerals that represent the solution of the task increase more rapidly than it would in a geometric series. This means probably that the «*presence of dots*» represents a factor that facilitates the impression of *vm, m, i*; that is, to have the impression of very many, many, indefinite, the necessary *objective* number (situation A) of dots is less than that assumed *by imagination* (situation B).

We may perhaps consider the importance, now realized, of the factor *presence of dots*, which one can call of «*perceptual vivacity or vividity*», as due to that impression of «imposition» that certain objects of perception may enliven in some subjects. One could perhaps find in the predisposition to particular *perceptual impressionabilities* one of the more important determinants of the under and overvaluator types.

3. – The density of amorphous complexes

We call «amorphous complex» a collectivity whose elements are disposed in such a way that they do not determine any particular impression of shape and cannot be grouped by any architectonic scheme. The dots are disposed homogeneously on a surface without giving any impression of figure. The surface on which they appear is constant (30 x 40 cm with the longer side vertical) and varies instead the objective number of dots and thus their density on the surface.

The duration of exposure is, in the present series, of 280 σ.

The task of the subjects (5 in this series) was to communicate in written form the numeral that more exactly corresponded to the im-

pression he had from the seen complex. The subject could also put in parenthesis another numeral corresponding, no longer to the first thoughtless evaluation, but to a successive evaluative elaboration. Twenty-three collectivities within the limits of 5 and 162 dots were exposed.

The obtained data may be summarized as follows.

1. – Subjects can be distinguished in undervaluators and overvaluators, being this a distinctive element that repeats itself in all experiments and that can be called «constitutional» in the subjects.

2. – The undervaluators have a slight tendency to overvaluate collectivities of less than 20 elements.

3. – The mean between under and overvaluators does not approach at all, except in the last part, the objective number. This results is particularly interesting for applied psychology; in fact it shows that a mean of evaluative data of this kind, obtained from different subjects, does not give a reliable testimony even if the data are inadequate in opposite directions and within large limits.

TABLE 4.

Obj. N.	M. underev. α	M. overev. β	$\frac{\alpha+\beta}{2}$
5	5.5	12	8.7
7	7.7	20	13.8
9	9.7	25	17.3
11	10.2	50	30.1
13	17	60	38.5
15	17	50	33.5
17	19.2	60	39.5
19	19.7	100	59.8
21	21.2	100	60.1
23	22	120	71
27	26.5	125	78.2
32	31	150	90.5
37	33.5	125	79.2
47	39.6	125	82.3
52	47.5	150	98.7
57	50.4	175	112.7
62	50	220	135
72	55	250	152.5
82	60.5	150	105.2
102	70.2	175	135
122	70	220	145
127	74.2	240	157
162	80	250	165

4. – Both the absolute undervaluation and overvaluation of the under and overvaluators increase with the number of elements.

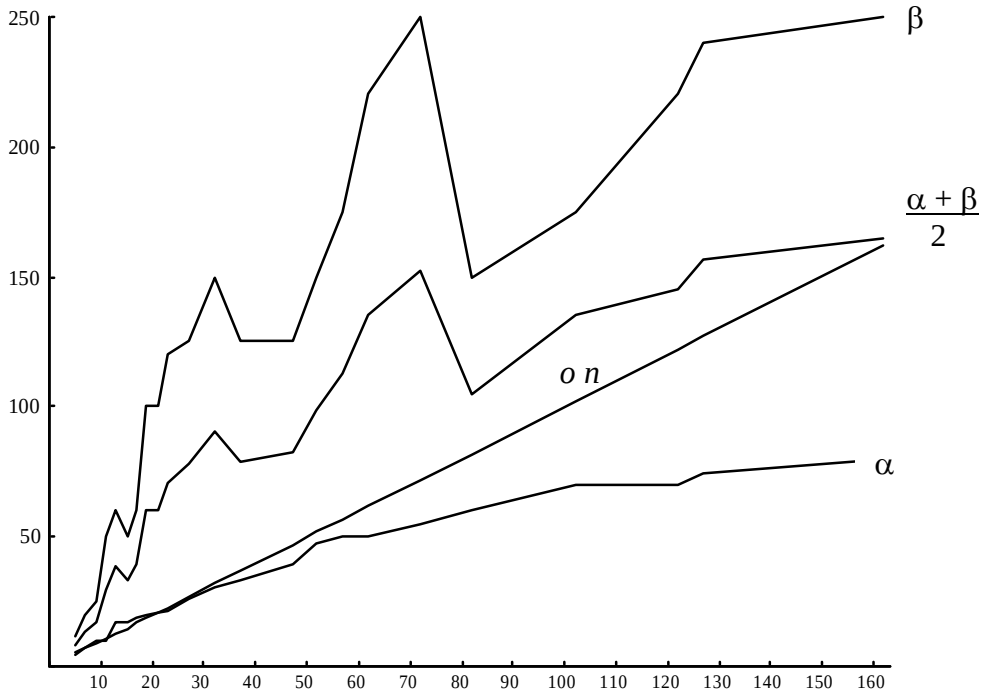

Fig. 9

I collect the obtained data in table 4. From this table I obtain the diagram of figure 9 in which the abscissa reports the objective number of dots and the ordinate reports the subjective evaluations. The curves α and β represent the mean subjective data of the undervaluator and overvaluator types, the mean of the two types, and the straight line *o n* the trend of what would be the adequate response.

We can note that, while the line α keeps an almost uniform trend, the line β shows very strong irregularities. Such diversities in trend are explained in part by the fact that the evaluative oscillations

of the undervaluators occur between necessarily narrow limits (in our case between 5 and 80) and those of the overvaluators between very large limits (between 12 and 250).

[*Translator's note*: the part of the text from this point of the original p. 196 to the end of the paper is not translated.]